Pre-Calculus/Trig 3 Name: ____________________________
UNIT 3: Exponents & Logarithms – MID UNIT REVIEW Date: ____________________________

GRAPHING EXPONENTIAL & LOGARITHMIC FUNCTIONS

Directions: Using the parent graph of $\boldsymbol{y = e^x}$, describe the transformations of each function.

1.) $f(x) = 2e^{x-8} - 5$ 2.) $f(x) = e^{2x+8} + 5$ 3.) $f(x) = -e^{x+2} + 2$

Directions: Using the parent graph of $\boldsymbol{y = \ln x}$, describe the transformations of each function.

4.) $f(x) = \ln\left(-\frac{1}{2}x + 2\right) - 6$ 5.) $f(x) = \frac{1}{3}\ln(x-2) - 3$ 6.) $4\ln(5x)$

Directions: Write the equation and/or the description of each function. Then graph the function if necessary.

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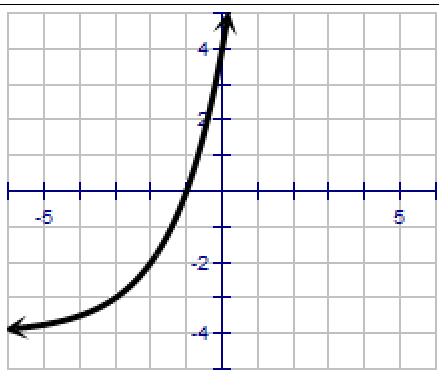
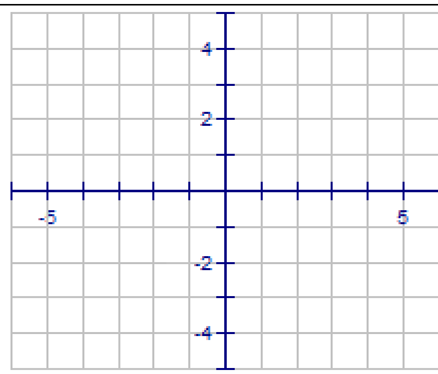
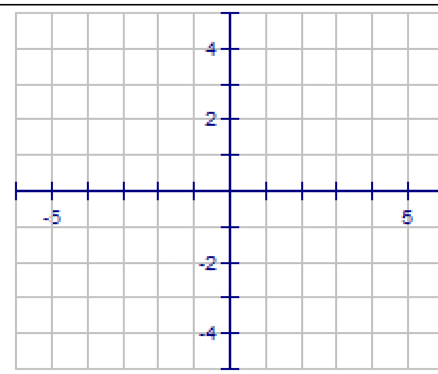
<tr><td></td><td></td><td></td></tr>
<tr><th>EQUATION</th><th>EQUATION</th><th>EQUATION</th></tr>
<tr><td>$f(x) =$ ____________________</td><td>$f(x) = 2^{x-1} + 1$</td><td>$f(x) =$ ____________________</td></tr>
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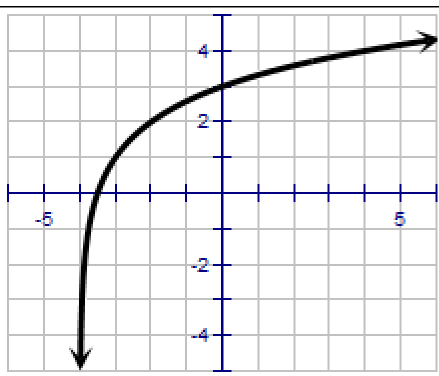
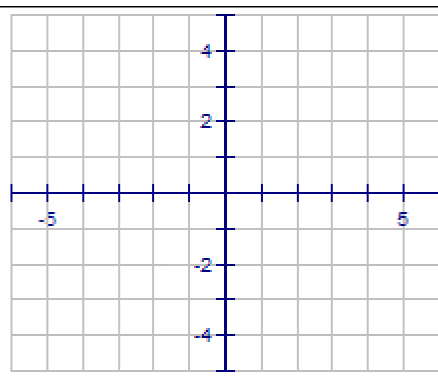
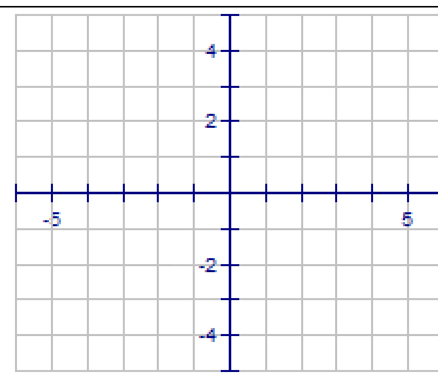
<tr><td></td><td></td><td></td></tr>
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REVIEW OF PROPERTIES OF LOGARITHMS

Directions: Write each exponential equation in logarithmic form.

1.) $2^4 = 16$ ____________	2.) $5^3 = 125$ ____________	3.) $3^{-2} = \frac{1}{9}$ ____________
4.) $7^0 = 1$ ____________	5.) $10^3 = 1000$ ____________	6.) $e^4 = 54.599$ ____________
7.) $8^{\frac{1}{3}} = 2$ ____________	8.) $25^{\frac{1}{2}} = 5$ ____________	9.) $2^{-1} = \frac{1}{2}$ ____________
10.) $e^{-2} = 0.1353$ ____________	11.) $10^{-2} = \frac{1}{100}$ ____________	12.) $3^{-3} = \frac{1}{27}$ ____________

Directions: Write each logarithmic equation in exponential form.

13.) $\log_2 32 = 5$ ____________	14.) $\log_9 81 = 2$ ____________	15.) $\log_2 \frac{1}{4} = -2$ ____________
16.) $\log_5 25 = 2$ ____________	17.) $\log_{16} 4 = \frac{1}{2}$ ____________	18.) $\log_{49} 7 = \frac{1}{2}$ ____________
19.) $\log 10 = 1$ ____________	20.) $\log_6 1 = 0$ ____________	21.) $\log_8 2 = \frac{1}{3}$ ____________
22.) $\ln 1 = 0$ ____________	23.) $\ln 5 = 1.609$ ____________	24.) $\ln \frac{1}{4} = -1.386$ ____________

Directions: Find the value of x that makes each logarithmic equation true.

25.) $\log_x 25 = 2$ ____________	26.) $\log_4 2 = x$ ____________	27.) $\log_{12} x = 1$ ____________
28.) $\log_6 x = 2$ ____________	29.) $\log_3 x = -1$ ____________	30.) $\log \frac{1}{100} = x$ ____________
31.) $\log_8 64 = x$ ____________	32.) $\log_9 1 = x$ ____________	33.) $\log_x \frac{1}{125} = -3$ ____________

Directions: Use properties of logarithms to evaluate each logarithmic expression.

34.) $\log_3 3^5 =$ ____________	35.) $\ln e^7 =$ ____________	36.) $\log 10^{-2} =$ ____________
37.) $6^{\log_6 15} =$ ____________	38.) $e^{ln 12} =$ ____________	39.) $10^{\log 4} =$ ____________
40.) $\ln e =$ ____________	41.) $\log_7 7 =$ ____________	42.) $\log 1 =$ ____________

Directions: Approximate each logarithm by using the *Change-of-Base Formula* to the nearest thousandth.

43.) $\log_7 19 =$ ______________ 44.) $\log_8 2 =$ ______________ 45.) $\log_4 0.75 =$ ______________

46.) $\log_6 12 =$ ______________ 47.) $\log_{15} 3 =$ ______________ 48.) $\log_2 0.125 =$ ______________

Directions: Expand each logarithmic expression. Do not leave any exponents in your final answer.

49.) $\log_3(9xy^2)$

50.) $\log\left(\frac{100x^2}{y^3}\right)$

51.) $\log_4\left(xy^2\sqrt{z}\right)$

52.) $\log_6(6x^2y)^3$

53.) $\log_5\left(\frac{x}{25y}\right)$

54.) $\log\left(\frac{10(x-1)^2}{yz^2}\right)$

Directions: Condense each logarithmic expression as a single logarithm.

55.) $2\log_3 x + 3\log_3 y - \log_3 z$

56.) $\frac{1}{2}[\log_2 x - (2\log_2 y + 3\log_2 z)]$

57.) $\log_4 4 + \log_4 x - \frac{1}{2}\log_4 y$

58.) $\log x - \log y - 2\log z$

59.) $2(\log x + 3\log y) - 5\log z$

60.) $\log_3 x - 2\log_3 y + 3\log_3 z$

APPLICATIONS & MODELS

INTEREST COMPOUNDED PERIODICALLY	INTEREST COMPOUNDED CONTINUOUSLY
$A = P\left(1 + \frac{r}{n}\right)^{nt}$	$A = Pe^{rt}$
EXPONENTIAL GROWTH	**EXPONENTIAL DECAY**
$y = ae^{bx}$	$y = ae^{-bx}$

PART 1: Find the interest and the amount for compounding periodically.

	BALANCE	EARNED INTEREST
1.) \$1,250 at 7% annually for 3 years	____________________	____________________
2.) \$23,600 at 5% semi-annually for 10 years	____________________	____________________
3.) \$5,000 at 12% monthly for 5 years	____________________	____________________
4.) \$51,275 at 6.5% quarterly for 8.5 years	____________________	____________________

PART 2: Find the interest and the amount for compounding continuously.

	BALANCE	EARNED INTEREST
5.) \$10,000 at 9% for 5 years	____________________	____________________
6.) \$240,000 at 7% for 25 years	____________________	____________________
7.) \$1,750 at 6.25% for 36 months	____________________	____________________
8.) \$17,625 at 4.5% for 7.5 years	____________________	____________________

PART 3: Apply the appropriate formula to solve the following compound interest application problems.

9.) The function describing the number of a rare birds that are found in a specific region after ***t months*** is given by:
$\boldsymbol{P(t) = 150e^{0.05t} \ \ where\ t \geq 0}$

a) What is the initial population of rare birds? Is this a situation of growth or decay?

b) What is the population of rare birds after 7 months?

10.) The population of a town is 50,000, and local authorities claim that the population is growing at an exponential rate of 4% per year, t.

a) Define the function that describes this situation: $P(t) =$ ________________________

b) Use your function to predict the population in 25 years?

ANNUITIES

PRESENT VALUE ANNUITY	FUTURE VALUE ANNUITY
$P_n = p\left[\frac{1-(1+i)^{-n}}{i}\right]$	$F_n = p\left[\frac{(1+i)^{n}-1}{i}\right]$

Directions: Answer each annuity application.

1.) ***IRA Scenario:*** Mrs. Jones is considering two different investment structures for her IRA.

Option #1: Pay $300 each month into an account with an APR of 4.2%.
Option #2: Pay $900 every quarter into an account with an APR of 4.3%.

(a) How much would she invest in each option after 10 years?

Option #1 = ________________

Option #2 = ________________

(b) Which option will give Mrs. Jones a better return on her investment in thirty years?

Option #1 = ________________

Option #2 = ________________

Best Option: ______________

(c) Which option will give Mrs. Jones a better return on her investment in five years?

Option #1 = ________________

Option #2 = ________________

Best Option: ______________

2.) ***Mortgage Scenario:*** Brian is looking to buy a home and has been given the following mortgage options:

Option #1: Offers a 30 year mortgage for $300,000 and APR 4.5% with monthly payments.

Option #2: Offers a 15 year mortgage for $275,000 and APR 4.75% with monthly payments.

(a) What will the monthly payment be for option #1? What is the total amount of interest paid over the life of loan in option #1? What is the total amount that Brian pays over the life of the loan in option #1?

Monthly payment: _______________

Total amount paid: _______________

Total interest paid: _______________

(b) What will the monthly payment be for option #2? What is the total amount of interest paid over the life of loan in option #2? What is the total amount that Brian pays over the life of the loan in option #2?

Monthly payment: _______________

Total amount paid: _______________

Total interest paid: _______________

(c) Which option should Brian choose? Why?

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